COMSOL Multiphysics: Surface-to-Surface Radiation Modeling Guide
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1 Introduction

This document acts as a guide of how to implement surface to surface radiation within COMSOL Multiphysics (5.0 and later) for case studies found within the built environment. Please note that this guide does not focus on implementing solar radiation models. This document, assumes that the reader has followed the course: 7LS9M0 Heat, Air & Moisture transfer/CFD1 (and additionally 7LS6M0 Heat, Air & Moisture transfer/CFD2) and therefore has some basic knowledge about COMSOL Multiphysics.

Within this guide, firstly a short step-by-step explanation is given on how to implement the radiation module within COMSOL. Beside these steps, several case studies are explained and compared to analytical solutions found within literature. Thirdly, also the calculations behind radiation modelling within COMSOL are theoretically explained. Fourthly, a brief discussion is added about the differences between the analytic solutions and model output.
2 Steps in modelling Surface-to-Surface Radiation

This guide assumes that a geometry (2D / 3D) is present, either with or without physics modules, studies and meshes. Some steps can be skipped, depending on the features already present in the model. To enter radiative heat transfer to the model in COMSOL Multiphysics 5.0, the following steps need to be taken:

1) **Modules:** Add the physics module: Heat Transfer with Surface-to-Surface Radiation (ht) [From: Heat Transfer > Radiation]

2) **Studies:** If no study is present, add the appropriate study (Steady-state / Time Dependent)

3) **Properties:** Add the material properties for conduction in solids (Heat capacity \([C_p]\), Density \([\rho]\), Conductivity \([\kappa]\)) : Either from the materials definition (assigning materials to the geometry) or by selecting “Heat transfer in solids” and choosing user-defined values in case no materials are defined.

4) **Initial values:** Select initial temperature values for the geometries [From: Heat Transfer with Surface-to-Surface Radiation (ht) > Initial values]. This is especially important for time-dependent studies, as this is the starting value for the first time step.

5) **Boundary conditions:** Add thermal boundaries to the geometry (Heat fluxes, Thermal boundaries, Thermal insulation) [Physics > Boundaries (condition)]

6) **Radiation boundary conditions:** Add surface properties for radiation to the geometry [Physics > Boundaries > Diffuse Surface]. Select a diffuse boundary for each geometry participating in radiation and assign the emissivity value for the surface. This can either be a user-defined value or a value allocated from earlier defined materials. Secondly, make sure that in the Diffuse Surface boundary, the “Include surface-to-surface radiation” option is checked, for each surface you want to include in the radiation study. In addition, define the ambient temperature \(T_{\text{amb}}\) for the boundary.

After defining the properties of the diffuse surface, the radiation settings need to be set up. The radiation direction is “opacity controlled” by default. This is applicable for all diffuse surfaces. For non-diffuse or transparent surfaces such as glass, the direction and wavelength-dependence needs to be investigated. The other options include: positive normal direction, negative normal direction and both sides which specifies the direction in which radiation can occur.

Important note, each surface which needs to be included within the surface to surface radiation (calculation) needs to have an emissivity value defined! For surfaces which are not defined, COMSOL will use by default an emissivity of 0. This means that this surface will not be affected by radiation. If a certain surface should act as a black body radiator, an emissivity value of 1 needs to be used.
A second important note is the definition of $T_{amb}$ for diffuse surfaces. $T_{amb}$ can be set to define far-away temperatures in directions where no other boundaries obstruct the view, such as radiation to the sky. Inside a closed volume, the ambient view factor, $F_{amb}$, is theoretically zero and the value of $T_{amb}$ therefore should not matter. However, if walls are defined as a 2D surface, radiation can still take place on the other side of the geometry, influencing temperature results. This can be avoided by adjusting the radiation direction, to only emit inside the volume, or by setting $T_{amb}$ equal to the indoor (air) temperature.

7) **Meshing:** If no mesh is present, create a mesh for the geometry. COMSOL uses the mesh facets to calculate the local view factors. For large surfaces, a course mesh is often sufficient; however, small discrepancies can be found at sharp corners or at surface edges. This is illustrated in figure 1, where course meshes are compared to fine meshes. [1]

![Figure 1](image)

**Figure 1:** Irradiation over three different 2D boundaries. Light lines represent course meshes while heavy lines represent a fine mesh [1]. Note that this figure is based on case study 0: Radiation in a triangular cavity with infinite length.

If the radiation is calculated from a temperature boundary with a non-uniform distribution, an exact temperature distribution at the radiating boundary is required to calculate the correct radiation fluxes. (This is often the case in models in which convection and radiation modules are combined). “The advancing front” meshing method for the “free triangular” setting can be used to get a dense mesh distribution at the boundary.
Figure 2: Advancing front mesh across conduction elements. Maximum element size at radiation boundary is 0.01 [2]. Note that this figure is based on case study 0: Radiation in a triangular cavity with infinite length.

8) **Post-processing:** After solving the model [model > compute] the radiation properties can be derived from the model. From the results, line integration profiles for e.g. temperatures, heat fluxes and surface radiosity can be obtained using expressions [Derived values > Line integration > Expression > Expression]. E.g. to define the boundary heat fluxes: [Replace expression > Component > Heat transfer with surface to surface radiation > Boundary fluxes > ht.ndflux – Normal conductive heat flux].
3 Case-studies

This section provides a brief description for different surface-to-surface radiation case studies. Each case focusses on a different configuration related to radiation heat transfer. The modelled results are compared to the analytical solution. Of these analytical solutions, the calculations are added in annex. The modelling files for each of these case are included in the folder. However, meshes and solutions have been cleared from these models to keep the file-size acceptable, and these have to be generated before the (correct) results can be displayed.
### 3.1 Case 0: Radiation in a triangular cavity with infinite length h\(^4\)

This case study is used by COMSOL to introduce people to the modelling of surface to surface radiation. Note that the exact guide for this tutorial can be found at their website \(^4\). Also note that \(^3\) also discusses this case study.

This geometry consists of three rectangles (3, 4 and 5 m and a width of 1 m) with the material properties of copper. The rectangles can only exchange heat through surface-to-surface radiation to one another.

**Input:** Heat flux and temperature boundaries, **Output:** surface-to-surface radiation flux and surface temperatures

<table>
<thead>
<tr>
<th>Model in- &amp; outputs</th>
<th>Unit</th>
<th>Object 1</th>
<th>Object 2</th>
<th>Object 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature T</td>
<td>K</td>
<td>300</td>
<td>641(^*)</td>
<td>600(^*)</td>
</tr>
<tr>
<td>Size (L x W )</td>
<td>m</td>
<td>4 x 1</td>
<td>3 x 1</td>
<td>5 x 1</td>
</tr>
<tr>
<td>Emissivity (\epsilon)</td>
<td>-</td>
<td>0.4</td>
<td>0.6</td>
<td>0.8</td>
</tr>
<tr>
<td>Heat flux Q</td>
<td>W/m(^2)</td>
<td>-</td>
<td>2000</td>
<td>1000</td>
</tr>
<tr>
<td></td>
<td>W/m</td>
<td>11,000(^*)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\(^*\)Analytical solution

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Analytical</th>
<th>COMSOL Multiphysics</th>
<th>Difference [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature Object 2 [K]</td>
<td>641.00</td>
<td>644.73</td>
<td>0.58</td>
</tr>
<tr>
<td>Temperature Object 3 [K]</td>
<td>600.00</td>
<td>600.86</td>
<td>0.14</td>
</tr>
<tr>
<td>Heat flux [W/m]</td>
<td>11,000</td>
<td>10,966</td>
<td>0.31</td>
</tr>
</tbody>
</table>
3.2 Case 1: Radiation between two infinitely long rectangular plates

This geometry consists of two rectangles of infinite length and width (modelled in 2D: length 1000 m, thickness 1 m, assumed an infinite width). The rectangles have prescribed temperatures

**Input:** Heat flux and temperature boundaries, **Output:** surface-to-surface radiation fluxes and solid temperatures (displayed in the table below).

Input:

Output:

<table>
<thead>
<tr>
<th>Unit</th>
<th>Object 1</th>
<th>Object 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature $T$</td>
<td>K</td>
<td>303,15</td>
</tr>
<tr>
<td>Size (L x W x H)</td>
<td>m</td>
<td>Infinite*</td>
</tr>
<tr>
<td>Distance surface to surface $h$</td>
<td>m</td>
<td>N.A.</td>
</tr>
<tr>
<td>Emissivity $ԑ$</td>
<td>-</td>
<td>0,9</td>
</tr>
<tr>
<td>Heat flux $Q_{12}$</td>
<td>W/m²</td>
<td>79.28*</td>
</tr>
</tbody>
</table>

* Analytical solution.
** Heat flux from object 1 to object 2 outcome calculated within literature.
pag 477. [3]
**Table:**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Analytical</th>
<th>COMSOL Multiphysics</th>
<th>Difference [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat flux [W/m²]</td>
<td>79.27</td>
<td>79.25</td>
<td>0.02</td>
</tr>
</tbody>
</table>

**Figure 3:** Radiative fluxes for both wall surfaces along-side the 1000 m long plate
Case 2: Radiation in a three dimensional rectangular enclosure

This geometry consists of a 1 x 1 x 1 m rectangular box. Two (non-opposing) walls have a temperature boundary condition applied to them, while the other sides consist of black body radiators ($\varepsilon = 1$).

**Input:** temperature boundaries, **Output:** surface-to-surface radiation fluxes and wall temperatures.

Two simulations were performed for this case: a high and low temperature case study:

- For the high temperature case the hot- and cold wall input temperatures are 1500 K and 500 K.
- For the low temperature case the hot- and cold wall input temperatures are 293.15 K and 323.15 K.

To model the walls of the box, the COMSOL model consists of a rectangular object for each side, to clearly separate the boundaries.

### High temperature case

<table>
<thead>
<tr>
<th>Unit</th>
<th>Hot wall</th>
<th>Cold wall</th>
<th>Black body surfaces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature T (K)</td>
<td>1493.3</td>
<td>506.66</td>
<td><strong>1247.50</strong></td>
</tr>
<tr>
<td>Size (L x W) (m)</td>
<td>1 x 1</td>
<td>1 x 1</td>
<td>1 x 1</td>
</tr>
<tr>
<td>Emissivity $\varepsilon$</td>
<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Heat flux (kW)</td>
<td><strong>-75.95</strong></td>
<td><strong>75.95</strong></td>
<td>N.A</td>
</tr>
</tbody>
</table>

### Low temperature case

<table>
<thead>
<tr>
<th>Unit</th>
<th>Hot wall</th>
<th>Cold wall</th>
<th>Black body surfaces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Temperature T (K)</td>
<td>322.15</td>
<td>293.35</td>
<td>308.44</td>
</tr>
<tr>
<td>Size (L x W) (m)</td>
<td>1 x 1</td>
<td>1 x 1</td>
<td>1 x 1</td>
</tr>
<tr>
<td>Emissivity $\varepsilon$</td>
<td>0.5</td>
<td>0.5</td>
<td>1.0</td>
</tr>
<tr>
<td>Heat flux (W)</td>
<td><strong>-53.75</strong></td>
<td><strong>53.75</strong></td>
<td>N.A</td>
</tr>
</tbody>
</table>

* Analytical solution.

---

**Figure 4:** Surface temperatures for the rectangular enclosure
Table 1: Case study 2: comparison analytical solution and model output

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Analytical</th>
<th>COMSOL Multiphysics</th>
<th>Difference [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>High temperature case:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Black body surface temp [K]:</td>
<td>1265.00</td>
<td>1247.50</td>
<td>1.38</td>
</tr>
<tr>
<td>Heat flux [kW]</td>
<td>77.40</td>
<td>75.95</td>
<td>1.87</td>
</tr>
<tr>
<td>Low temperature case:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Black body surface temp [K]:</td>
<td>309.24</td>
<td>308.44</td>
<td>0.27</td>
</tr>
<tr>
<td>Heat flux [W]</td>
<td>54</td>
<td>53.75</td>
<td>0.46</td>
</tr>
</tbody>
</table>
3.4 Case 3: Radiation in an ice rink building

This geometry consists of a 50 by 10 m 2D ice-skate hall. Two side walls have a temperature boundary condition of 15°C. The ice surface has a temperature boundary of -5°C. The ceiling is a grey body with two possible paint layers ($\varepsilon = 0.05$ or $\varepsilon = 0.94$), while the other sides consist of black-body radiators ($\varepsilon = 1$). A convective heat flux of 5 W/m²K was applied to the lower ceiling boundary to account for convective losses to the air. 

**Input:** temperature boundaries/convective heat-fluxes, surface properties **Output:** surface temperatures

Two simulations were performed for this case, one for each of the two paint layers:  
High reflectivity, the ceiling emissivity $\varepsilon = 0.05$.  
Low reflectivity, the ceiling emissivity $\varepsilon = 0.94$.  

**SCHEMATIC:**

![Diagram of ice rink building with specified conditions and properties.]

Figure 5: Input values for the ice rink building [6]  

Figure 6: Output isotherms for the low reflectivity case (left) and high reflectivity case (right)
Figure 7: Output temperature along the ceiling of the ice rink building for the low- and high emissivity case

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Analytical</th>
<th>COMSOL Multiphysics</th>
<th>Difference [%]</th>
</tr>
</thead>
<tbody>
<tr>
<td>High reflectivity case:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Black body surface temp [K]:</td>
<td>287.2</td>
<td>287.0</td>
<td>0.06</td>
</tr>
<tr>
<td>Low reflectivity case:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Black body surface temp [K]:</td>
<td>281.7</td>
<td>280.4</td>
<td>0.46</td>
</tr>
</tbody>
</table>
4 Theory

COMSOL computes the total radiative flux leaving a surface (radiosity). This approach is valid only for isothermal surfaces and objects. This means that the radiative properties (emissivity \( \varepsilon \), absorptivity \( \alpha \), reflectivity \( \rho \)) remain constant. This applies to cases where \( \alpha = \varepsilon \). These conditions are satisfied for: i) cases in which the irradiation is diffuse, ii) the surface is diffuse, iii) the radiation spectrum only depends on the temperature of the object (black-body radiation) or iv) if the radiation properties are independent of the wavelength. Overall, the radiosity can be defined as:

\[
Q_{ij} = A_i F_{ij} (J_i - J_j)
\]

Where:
- \( Q_{ij} \) = the energy transmitted from body i to body j [W]
- \( A_i \) = the surface area of body i \([m^2]\)
- \( F_{ij} \) = the view factor from body i to body j [\(-\)]
- \( J_i \) = total radiative flux leaving surface i (radiosity) \([W/m^2]\)
- \( J_j \) = total radiative flux leaving surface j (radiosity) \([W/m^2]\)

For a certain temperature difference between two surfaces the transmitted radiation from surface i can be defined as:

\[
Q_i = \frac{A_i \varepsilon_i (\sigma T_i^4 - J_i)}{1 - \varepsilon_i}
\]

Where:
- \( Q_i \) = the thermal energy leaving surface i [W]
- \( \varepsilon_i \) = the emissivity of surface i \([m^2]\)
- \( \sigma \) = the Stefan-Boltzmann constant \((5.670373 \times 10^{-8})\) [J/s/m²/K⁴]
- \( T_i \) = the temperature of surface i [K]

This means that the radiation flux between two surfaces of different temperature can be expressed by:

\[
Q_i = \sigma A_i \varepsilon_i (T_i^4 - T_j^4)
\]

Where:
- \( T_i \) = the temperature of surface i [K]
- \( T_j \) = the temperature of surface j [K]

In COMSOL 5.0 four radiation modules are included. This study focuses on the module: “heat transfer with Surface-to-Surface radiation (ht)” (from: Physics -> Heat Transfer -> Radiation -> heat transfer with Surface-to-Surface radiation(ht)). In the application area of surface to radiation in the built environment, the constraints for \( \alpha = \varepsilon \), and constant radiative properties are satisfied.

As can be seen from equation (1) the view factor (F) plays an important role within the radiation transfer. The view factor is only dependent on the geometry of the radiating bodies and can be seen as the fraction of radiation from surface i that is intercepted by surface j. It can be expressed as:

\[
F_{ij} = \frac{\text{diffuse energy leaving } A_i \text{ directly toward and intercepted by } A_j}{\text{total diffuse energy leaving } A_i}
\]
In COMSOL the surface is often divided into a large number of sub-surfaces by meshing the geometry, these sub-surfaces are referred to as facets. The view factor is automatically determined in COMSOL for each of the facets using a “hemicube” or “direct area integration” approach.

The view factors can also be extracted from COMSOL (as discussed in the tutorial: view factor computation). However in this version of COMSOL (5.0) there is no functionality to manually input the view factors for the calculation (in case these are known in advance).

5 Discussion

The deviation values are calculated based on the absolute values. This allows for a honest comparison between de high and low temperature cases. However, for cases with small temperature differences in [K], the accuracy might be much lower for comparing temperature differences.

Radiative fluxes are very accurately estimated by COMSOL as becomes clear from the validation cases. For the temperatures; however, accuracy is significantly lower. There can be a few reasons why this is be the case: the analytical solutions computes an average value for an entire boundary, while COMSOL calculates location specific values. In addition, the effects in sharp corners, as explained in chapter 2, also causes some differences for the area integrated averages. The difference in temperature predictions was most noticeable in the ice rink case (case 3) where estimations for the low reflectivity case, were significantly more accurate than for the high reflectivity case. This would indicate that the more uniform temperature distributions for the low ε is easier to predict than the curved temperature profile encountered for the high ε case. This should be taken into account when modeling surface to surface radiation using COMSOL Multiphysics.

Nevertheless, in cases where indoor environments are modeled the radiative heat transfer is often only one aspect of heat transfer, as convective and conductive heat transfer are involved as well. Including these will likely lead to even more precise predictions of building surface temperatures.

6 References


[3]: Research and Education Association, Fogiel M. (1999), The Physics Problem Solver Heat transfer, a complete solution guide to any textbook (Vol. 3). Research & Education Association


Appendix A: Analytical solution for case 0. [1]

appears in the section “Enclosures with More Than Two Surfaces” in Ref. 1.

![Diagram of a triangular cavity with labels L_1, L_2, L_3, ε_1, ε_2, ε_3, Q_2, and Q_3.]

*Figure 5: Problem sketch for theoretical analysis of radiation in a triangular cavity.*

The theoretical model considers only the three boundaries that form the cavity. On these boundaries, the model either holds the temperature at a constant value or specifies a heat flux. Note that this approach differs somewhat from the model, which sets temperatures and heat fluxes on the rectangles’ outer boundaries.

The COMSOL Multiphysics model shows that the temperatures and heat fluxes are nearly equal on the inner and outer boundaries of the rectangles. Therefore, the COMSOL Multiphysics and the theoretical model show good agreement.

Using the notation from *Figure 5* with the same assumptions as for the model, you obtain the following lengths and emissivities:

\[ L_1 = 4 \text{ m}, \quad \varepsilon_1 = 0.4 \]
\[ L_2 = 3 \text{ m}, \quad \varepsilon_2 = 0.6 \]
\[ L_3 = 5 \text{ m}, \quad \varepsilon_3 = 0.8 \]

The boundary conditions define either the temperature or the heat flux on each boundary. Now apply the same boundary conditions as in the problem where you set values for \( T_1, Q_2, \) and \( Q_3 \). You must, however, make a small adjustment for \( T_1 \) because the theoretical configuration sets it on the outer boundary and not on the cavity side. \( T_1 \) is therefore slightly higher than 300 K in the theoretical analysis:
\[ T_1 = 307 \text{ K} \]
\[ Q_2 = q_2 L_2 = 2000 \frac{\text{W}}{\text{m}} \times 3 \text{ m} = 6000 \text{ W} \]
\[ Q_3 = q_3 L_3 = 1000 \frac{\text{W}}{\text{m}} \times 5 \text{ m} = 5000 \text{ W} \]

You describe the heat flux from a boundary with the following two equations:

\[ Q_i = L_i \frac{\varepsilon_i}{1 - \varepsilon_i} (\sigma T_i^4 - J_i) \]
\[ Q_i = L_i \sum_k F_{ik} (J_i - J_k) \]

For a triangular cavity, the following equation gives the view factor between surface \( i \) and surface \( j \):

\[ F_{ij} = \frac{L_i + L_j - L_k}{2L_i} \]

Substituting this expression for the view factor into the second equation for the heat fluxes results in a linear system of six equations. The six unknowns for this particular problem are \( J_1, J_2, J_3, Q_1, T_2, \) and \( T_3 \), and you would like to compare the three last to those from the solution. Solving the linear system yields the following values: \( Q_1 = -11,000 \text{ W/m}, T_2 = 641 \text{ K}, \) and \( T_3 = 600 \text{ K} \).

To compare these results with the model, you must find the total heat flux through the cavity’s bottom (horizontal) boundary and also calculate the average temperatures on the other two boundaries.

The following table compares the results from COMSOL Multiphysics with the theoretical values:

<table>
<thead>
<tr>
<th>QUANTITY</th>
<th>COMSOL MULTIPHYSICS</th>
<th>THEORY</th>
</tr>
</thead>
<tbody>
<tr>
<td>( Q_1 )</td>
<td>-10,965 W/m</td>
<td>-11,000 W/m</td>
</tr>
<tr>
<td>( T_2 )</td>
<td>645 K</td>
<td>641 K</td>
</tr>
<tr>
<td>( T_3 )</td>
<td>601 K</td>
<td>600 K</td>
</tr>
</tbody>
</table>

The differences are quite small. Also note that the theoretical model includes some simplifications. For example, it assumes that heat fluxes and temperatures are constant along each boundary. It also assumes a constant view factor for each pair of boundaries,
Appendix B: Analytical solution for case 1. [3]

(Units were converted to the metric system.)

Compute the rate of radiation heat transfer between two large parallel plates having the temperatures and emissivities as shown in the figure. Also, compute the rate of radiation heat transfer, if a radiation shield, emissivity 0.06, is provided inbetween the walls.

Parallel plates.

**Solution:** a) The radiation heat flux is expressed as

\[ q = A\sigma(T_1^a - T_2^a) \text{ Btu/hr} \]

where \( A \) is the surface

- Stefan-Boltzmann constant -- \( 0.171 \times 10^{-8} \)
- \( T \)-absolute temperature in Rankine

\[ f = \frac{1}{1/T_1 + 1/T_3} - 1 \]

\[ = \frac{1}{0.9 + 0.4} - 1 \]

\[ = 0.383 \]

\[ \therefore q_A = 0.383 \times 0.171 \times 10^{-8} \ \text{[(120 + 460)°} - (70 + 460)°]\]

\[ = 22.49 \text{ Btu/hr-ft}^2 \]

Again, with a shield present between the surfaces, for equilibrium conditions,

\[ q = \frac{\sigma(T_1^a - T_2^a)}{1/ε_1 + 1/ε_2 - 1} = \frac{\sigma(T_2^a - T_1^a)}{1/ε_2 + 1/ε_3 - 1} \]

4??
Introducing the known values, the temperature of the shield can be found.

\[
\begin{align*}
T_1^4 - T_2^4 &= \frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1 = \frac{1}{0.9} + \frac{1}{0.06} - 1 \\
T_2^4 - T_3^4 &= \frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_3} - 1 = \frac{1}{0.66} + \frac{1}{0.4} - 1
\end{align*}
\]

\[
T_1^4 - T_2^4 = 0.92(T_3)^4 - 0.92(T_3)^4
\]

or

\[
1.92 T_2^4 = T_1^4 + 0.92 (T_3)^4
\]

\[
(T_2)^4 = \frac{1((120 + 460)^4 + 0.92(70+460)^4)}{1.92}
\]

\[
(T_2)^4 = 9.67 \times 10^{19} \Rightarrow T_2 = 550^\circ R(98^\circ C)
\]

The radiation heat transfer rate with a shield present is

\[
\frac{Q}{A} = \frac{(T_1^4 - T_2^4)}{\frac{1}{\varepsilon_1} + \frac{1}{\varepsilon_2} - 1}
\]

\[
= \frac{0.171 \times 10^{-9} [(580)^4 - 9.7 \times 10^{16}]}{0.9 + 0.06 - 1}
\]

\[
= 1.65 \text{ Btu/hr-ft}^2
\]

The presence of a radiation shield drastically reduced the net rate of radiation heat transfer.
Appendix C: Analytical solution for case 2. [3]
(The same methodology was followed for the low temperature case)

One side of a cubical box, side 1m, is generating heat at 1227°C. The adjacent heat-absorbing side is at 227°C. The emissivity for both sides is 0.5. Except the generating and the absorbing black sides, all the sides are gray surfaces. Compute the temperature at the gray surfaces, also the net radiant heat exchange.

Solution: Figure 2 is the network representation of the problem. There are three surfaces in the radiation exchange— one is the heat-generating face (1), another is the heat-absorbing face (2) and third is the gray surface(s) (3).
Considering the symmetricity of the network, the radiosity of the surface 3 can be computed as the average of the radiosities of surfaces 1 and 2, and then from the radiosity of surface 3 the temperature of the gray surfaces can be determined.

Radiosity of surface 1
\[ B_1 = 0(T_1)^b = 5.67 \times 10^{-8} (1227 + 273)^b \]
\[ = 5.67 \times 10^{-8} (1500)^b \]
\[ \equiv 287040 \text{ W/m}^2 \]

\[ B_2 = \sigma T_2^4 = 5.67 \times 10^{-8} (227 + 273)^4 \]
\[ = 5.67 \times 10^{-8} (500)^4 \]
\[ = 3540 \text{ W/m}^2 \]

\[ B_3 = \frac{B_1 + B_2}{2} = \frac{287040 + 3540}{2} \]
\[ = 145290 \text{ W/m}^2 \]

or

\[ B_1 = \frac{T_1^4}{B_3} \]
\[ \therefore T_3 = \frac{145290}{5.67 \times 10^{-8}} = \frac{145290}{0.057} \]
\[ \approx 2535^\circ \text{K} \ (992^\circ \text{C}) \]

CLOSED-FORM EXPRESSIONS OF SELECTED SHAPE FACTORS

<table>
<thead>
<tr>
<th>Geometry</th>
<th>Expression</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Opposite rectangles</td>
<td>[ f_{1-2} = \frac{1}{2} \frac{x}{a} \sqrt{\frac{(1-x^2)}{(1-x^2)^2 + \frac{b}{c}x}} ]</td>
</tr>
<tr>
<td>2. Adjacent rectangles</td>
<td>[ f_{1-2} = \frac{1}{2} \frac{x}{a} \sqrt{\frac{4x(1-x^2)}{3(1-x^2) + 4(1-x^2)^2}} ]</td>
</tr>
</tbody>
</table>

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Again, from the table, the shape factor $F_{1-2}$ for the adjacent faces is expressed as

$$F_{1-2} = \frac{1}{4\pi x} \left[ 4x\tan^{-1} \frac{1}{x} + 4y\tan^{-1} \frac{1}{y} \right]
-4 \left( x^2 + y^2 \right) \tan^{-1} \frac{1}{x^2 + y^2}
+ 2\ln \frac{(1 + x^2)(1 + y^2)}{1 + x^2 + y^2}
+ x^2 \ln \frac{x^2(1 + x^2 + y^2)}{(1 + x^2)(x^2 + y^2)}
+ y^2 \ln \frac{y^2(1 + x^2 + y^2)}{(1 + y^2)(x^2 + y^2)}$$

where $x$ and $y$ are the ratios of the sides of the faces.

$x = \frac{a}{c} = \frac{1}{1} = 1$

$y = \frac{b}{c} = \frac{1}{1} = 1$

$x = y = 1$, then,

$$F_{1-2} = \frac{1}{4\pi(1)} \left[ 4\tan^{-1}(1) + 4 \tan^{-1}(1) - 4 \left( 1^2 + 1 \right) \right]
-4 \left( 1 + 1^2 \right) \tan^{-1} \frac{1}{1 + 1^2}
+ \ln \frac{(1 + 1^2)(1 + 1^2)}{1 + 1 + 1}
+ 1^2 \ln \frac{1^2(1 + 1^2 + 1^2)}{(1 + 1^2)(1^2 + 1^2)}
+ 1^2 \ln \frac{1^2(1 + 1^2 + 1^2)}{(1 + 1^2)(1^2 + 1^2)}$$

Simplifying,

$$F_{1-2} = \frac{1}{4\pi} \left[ 8\tan^{-1}(1) + 4\frac{\sqrt{2}\tan^{-1} \left( \frac{1}{\sqrt{2}} \right)}{3} + 2\ln \frac{3}{2} \right]$$

$= 0.2$

$F_{1-3} = F_{2-3} = (1 - 0.2) = 0.8$ and

$F_{1-1} = F_{2-2} = 0$

$F_{1-3} = 0.8$ shows that the 80% of the radiation from the heat generating surface 1 falls on surface 3, i.e. grey surface. 50% of this 80%, i.e., 40% of the radiation leaving surface 1 falls back to it. $F_{1-2} = 0.2$ indicates that only 20% of the radiation falls on the heat absorbing surface 2.

The equivalent conductance of the gray surfaces can be evaluated as from the expression
\[ A_{1}f_{1-2, b} = \frac{1}{\frac{1}{A_{1}F_{1-3}} + \frac{1}{A_{2}F_{2-3}}} \]

\[ A_{1} = A_{2} = 1 \text{m}^2 \]

\[ \therefore f_{1-2, b} = \frac{0.2 + \frac{1}{0.8 + 0.8}}{0.6} = 0.6 \]

The subscript \( b \) refers to black body.

Now,

\[ f_{1-2} = \frac{1}{f_{1-2, b} + \frac{1 - \varepsilon_{2}}{\varepsilon_{1}} + \frac{1 - \varepsilon_{2}}{\varepsilon_{1}}} \]

\[ = \frac{1}{0.6 + 1 - 0.5 + 1 - 0.5} = 0.273 \]

The net exchange of heat flux is given by

\[ Q = A_{1}f_{1-2} \sigma(\text{T}_1^4 - \text{T}_2^4) \]

\[ = 1 \times 0.273 \times 5.67 \times 10^{-8} \times (1500^4 - 500^4) \]

\[ = 77396 \text{ W} \text{ or } \approx 77.4 \text{ kW} \]
Appendix D: Analytical solution for case 3. [6]  
(The same methodology was followed for the low temperature case)

Problem 13.93: Assessment of ceiling radiative properties for an ice rink in terms of ability to maintain surface temperature above the dew point.

KNOWN: Ice rink with prescribed ice, rink air, wall, ceiling and outdoor air conditions.

FIND: (a) Temperature of the ceiling, $T_c$, for an emissivity of 0.05 (highly reflective panels) or 0.94 (painted panels); determine whether condensation will occur for either or both ceiling panel types if the relative humidity of the rink air is 70%, and (b) Calculate and plot the ceiling temperature as a function of ceiling insulation thickness for $0.1 \leq t \leq 1$ m; identify conditions for which condensation will occur on the ceiling.

PROPERTIES: Psychometric chart (Atmospheric pressure; dry bulb temperature, $T_{db} = T_{\infty,i} = 15^\circ$C; relative humidity, RH = 70%): Dew point temperature, $T_{dp} = 9.4^\circ$C.
ANALYSIS: Applying an energy balance to the inner surface of the ceiling and treating all heat rates as energy outflows,

\[ E_{in} - E_{out} = 0 \]

where the rate equations for each process are

\[ q_o = \left( T_c - T_{\infty,o} \right) / R_{cond} \]
\[ R_{cond} = t / kA_c \]

\[ q_{conv,c} = h_i A_c \left( T_c - T_{\infty,i} \right) \]
\[ q_{rad,c} = \varepsilon E_b \theta c \cdot \theta c - \alpha A_w F_{wc} E_b \theta w \cdot \theta w - \alpha A_i F_{ic} E_b \theta i \cdot \theta i \]

Since the ceiling panels are diffuse-gray, \( \alpha = \varepsilon \).

From Table 13.2 for parallel, coaxial disks

\( F_{ic} = 0.672 \)

From the summation rule applied to the ice (i) and the reciprocity rule,

\[ F_{ic} + F_{iw} = 1 \quad F_{iw} = F_{cw} \text{ (symmetry)} \]
\[ F_{cw} = 1 - F_{ic} \]
\[ F_{wc} = \frac{\theta c}{A_w} \quad \theta c = \frac{\theta c}{A_w} - F_{ic} = 0.410 \]

where \( A_c = \pi D^2/4 \) and \( A_w = \pi DL \).

Using the foregoing energy balance, Eq. (1), and the rate equations, Eqs. (2-5), the ceiling temperature is calculated using radiative properties for the two panel types,

<table>
<thead>
<tr>
<th>Ceiling panel</th>
<th>( \varepsilon )</th>
<th>( T_c ) (°C)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflective</td>
<td>0.05</td>
<td>14.0</td>
</tr>
<tr>
<td>Paint</td>
<td>0.94</td>
<td>8.6</td>
</tr>
</tbody>
</table>

Condensation will occur on the painted panel since \( T_c < T_{dp} \).

(b) Applying the foregoing model for \( 0.1 \leq t \leq 1.0 \) m, the following result is obtained

\[ \text{Ceiling temperature, } T_c \text{ (°C)} \]

\[ \text{Ceiling insulation thickness, } t \text{ (m)} \]

\[ \text{Painted ceiling, } \varepsilon_{sc} = 0.94 \]
\[ \text{Reflective panel, } \varepsilon_{sc} = 0.05 \]