1 Problem Specification

Often we are only interested in the total heat flux through a plain structure, even though the flux is not evenly distributed, because of e.g. thermal bridges. We want to know the equivalent thermal transmittance for the whole structure, which can be defined as:

The total heat flow divided by the temperature drop over the construction per square meter of surface area [W/m².K].

This coefficient can be estimated by considering physically two extreme situations that give an upper and a lower limit to this coefficient. These extreme situations can be found by hand calculations when we simplified the problem. This will be elaborated in the method section.

The goal of this study is to discover the equivalent thermal conductivity of a plane structure with thermal bridges by both simulation and estimation using hand calculations. Also visualization and explanation of both methods have a high priority. This study is a two-dimensional steady state problem with no fluid flow or source terms.

The energy transport equation in a one dimensional setting is:

\[
\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} = \frac{\lambda}{\rho \cdot c_p} \frac{\partial^2 T}{\partial x^2} + \frac{Q}{\rho \cdot c_p}
\]

The only mode of heat transfer in this study is by diffusion, so the problem is a 2D steady state diffusion problem. The aforementioned equation changes to:

\[
0 = \frac{k}{\rho \cdot c_p} \cdot \nabla^2 T
\]
2 Method

In this study we are looking for the thermal conductivity properties of a 2D floor construction model which is described in the ‘Model’ section. To do this, we will use the simulation software program COMSOL Multiphysics v3.5, and some formulas, which we use for estimating the equivalent thermal transmittance by hand calculations.

With COMSOL we get information about the heat flow and temperature distribution through the structure.

By hand calculations the equivalent thermal transmittance of a plane structure with thermal bridges can be estimated by considering two extreme situations. These situations are:

1. minimal heat transmittance

   In this case we make the assumption there is no lateral heat exchange, as if ideal insulating layers are applied parallel to the heat flow, between different material structures perpendicular to the interested heat flow. All the heat flow lines in this scenario are straight and perpendicular to the surface. This minimum can be calculated using the following equation:

   \[ U_{\text{min}} = \frac{1}{A} \sum_k \frac{A_k}{R_i + R_e + \sum_j R_{kj}} \]

2. maximum heat transmittance

   The upper limit of thermal transmittance will be reached when maximal lateral heat exchange is assumed between the material layers as if ideal conducting thin layers parallel to the surface are applied. In this scenario all isotherms will be straight and run parallel to the surface.

   \[ U_{\text{max}} = \frac{1}{R_i + R_e + \sum_j \frac{A_k}{\sum_k R_{kj}}} \]

   The equivalent thermal transmittance \( U_{\text{eq}} \) is somewhere between \( U_{\text{min}} \) and \( U_{\text{max}} \). An estimation is given by:

   \[ \frac{1}{U_{\text{gem}}} = 0.5 \cdot \left( \frac{1}{U_{\text{min}}} + \frac{1}{U_{\text{max}}} \right) \]

   We used COMSOL to simulate two adapted structures of the real situation. This imitates the situations in which the physical maximum and minimum of possible heat exchange should take place by using thin super conductors and insulators.
3 The model

Geometry and mesh properties

Dimensions [m]:

<table>
<thead>
<tr>
<th>Component</th>
<th>Dimensions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wooden floor (WxH)</td>
<td>0,50x0,05</td>
</tr>
<tr>
<td>Wooden ceiling (WxH)</td>
<td>0,50x0,05</td>
</tr>
<tr>
<td>Steel beams(WxH)</td>
<td>0,05x0,15</td>
</tr>
<tr>
<td>Insulation material(WxH)</td>
<td>0,40x0,15</td>
</tr>
<tr>
<td>Thickness super insulators and conductors</td>
<td>0,002 (only in adapted models)</td>
</tr>
</tbody>
</table>

Fig. 1 shows the three 2D-models which we have studied. The upper model represents the real structure which consist of a wooden floor on the top and a wooden ceiling on the bottom with insulation material within. The steel beams that connect the wooden structures will act as thermal bridges through the insulation layer.

The second model shows how the original model is adapted with super conductors to reach the maximum physical heat flux through the construction as, we also estimate in our hand calculations.

The third model shows the adapted model with super insulators, which we use to discover what the minimum physical heat flux through the construction is, as we also assume in our hand calculations.
Fig. 1 Visualization of the simulated models
Mesh settings:
The mesh of the three constructions together consists of 33,289 elements.
Below the mesh geometry and statistics of the model are presented:

![Mesh geometry and statistics](image)

**Boundary Conditions**
- Air temperature above construction = 10°C
- Air temperature below construction = 25°C
- Surface heat transfer coefficients = 7.7 W/m².K (top and bottom)
- Thermal insulation boundaries = left and right side boundaries
Material Properties and Initial Condition

**Thermal conductivity k:**

- Wooden elements: $0.4 \text{ [W/m.K]}$
- Insulation: $0.04 \text{ [W/m.K]}$
- Steel beams: $45.0 \text{ [W/m.K]}$
- Super conductor: $100,000.0 \text{ [W/m.K]}$
- Super insulator: $0.00001 \text{ [W/m.K]}$

**Initial temperature conditions:** $20 \text{ [ºC]}$

Fig. 3 and 5 shows the adapted construction model with super conductors and respectively super insulators. The colors represent the thermal conductivity of each material. Due to the large scale caused by e.g. super conductors there is no color distinction between the other materials with relative small thermal conductivity difference.
Solver Settings

Type of analysis: Stationary Heat Transfer by conduction (ht)
Linear system solver: Direct (UMFPACK)
Automatic Matrix symmetry
Pivot threshold: 0.1
Memory allocation factor: 0.7

Fig. 4 Simulation model of construction with super insulators.
4 Results

Simulation results

Below the temperature distribution through the constructions are visualized. At the right side of the models the simulated mean heat flux and thermal resistance are shown:

- **Real Construction**: Mean Heat flux: 12.86 W/m², Mean Thermal Resistance: 1.17 m².k/W
- **Construction with super conductors**: Mean Heat flux: 28.5 W/m², Mean Thermal Resistance: 0.35 m².k/W
- **Construction with super insulators**: Mean Heat flux: 8.52 W/m², Mean Thermal Resistance: 1.76 m².k/W

*Fig. 5 Simulated temperature in the real and adapted constructions under stationary conditions*
Fig. 6 shows the isotherms and heat flux lines in the simulated models:

**Real Construction**

**Construction with super conductors**

**Construction with super insulators**
Calculation results

With hand calculations described in the method section we can calculate the minimum and maximum limits of total heat flux through the plane structure with thermal bridges. Out of these results we calculate the equivalent thermal transmittance by middle the two limits.

Surface ratio between the steel beams and insulation material is:

\[ f_A = \frac{A_{\text{beams}}}{A_{\text{tot}}} = \frac{0.1}{0.5} = 0,2 \]

The maximum thermal conductivity in case of short circuit between the layers:

\[ U_{\text{max}} = \frac{1}{\sum_j \left( \frac{f_A}{R_{kj}} + \frac{1-f_A}{R_{cj}} \right) + R_i + R_e} \]

\[ U_{\text{max}} = \frac{1}{\left( \frac{0.2}{0.05} + \frac{1-0.2}{0.05} + \frac{0.2}{0.15} + \frac{1-0.2}{0.15} + \frac{0.2}{0.05} + \frac{1-0.2}{0.05} \right) + \frac{1}{7.7} + \frac{1}{7.7}} \]

\[ U_{\text{max}} = 1,90 \text{W/m}^2.\text{k} \]

The minimum thermal conductivity in case of vertical separation of different construction compositions, which results in only one-dimensional heat fluxes:

\[ U_{\text{min}} = \frac{f_A}{\sum R_{kj} + R_i + R_e} + \frac{1-f_A}{\sum R_{cj} + R_i + R_e} \]

\[ U_{\text{min}} = \frac{0,2}{\left( \frac{0.05}{0.4} + \frac{0.15}{0.4} + \frac{0.05}{0.4} \right) + \frac{1}{7.7} + \frac{1}{7.7}} + \frac{0.8}{\left( \frac{0.05}{0.4} + \frac{0.15}{0.4} + \frac{0.05}{0.4} \right) + \frac{1}{7.7} + \frac{1}{7.7}} \]

\[ U_{\text{min}} = 0,58 \text{W/m}^2.\text{k} \]

So the average thermal transmittance between these limits is:

\[ \frac{1}{U_{\text{av}}} = 0.5 \cdot \left( \frac{1}{U_{\text{min}}} + \frac{1}{U_{\text{max}}} \right) = 0.5 \cdot \left( \frac{1}{0.578} + \frac{1}{1.90} \right) = 1,13 \text{m}^2.\text{k/W} \]

\[ U_{\text{av}} = 0.89 \text{W/m}^2.\text{k} \]
5 Conclusion

Comparing the results between the simulations and hand calculations, we can conclude that these are in good agreement. The equivalent thermal transmittance, which we have approached by taking the average out of the two limits obtained by hand calculations, also match with the simulation results of the real construction. This means that the real thermal conductivity of this construction is indeed approximately the average between the minimal and maximal calculated limits.

Bibliography

M.h. de Wit, [2009]: Heat, air and moisture in building envelopes. Course book Eindhoven University of Technology, blz 39 – 42.